রচনামূলক প্রশ্ন ও উত্তর

1. $\dfrac{6x-3}{(x+1)^2(x-2)}$ কে আংশিক ভগ্নাংশে প্রকাশ কর।

সমাধান: মনেকরি, $\dfrac{6x-3}{(x+1)^2(x-2)} \equiv \dfrac{A}{x+1} + \dfrac{B}{(x+1)^2} + \dfrac{C}{x-2}$ --- (i)

(i) নং-এর উভয় পক্ষকে $(x+1)^2(x-2)$ দ্বারা গুণ করে—

$$6x-3 \equiv A(x+1)(x-2) + B(x-2) + C(x+1)^2 \text{ ---- (ii)}$$

(ii) নং-এ $x=-1$ বসিয়ে

$$6\times(-1) - 3 = A(-1+1)(-1-2) + B(-1-2) + C(-1+1)^2$$

$$\Rightarrow -9 = B\times(-3)$$

$$\Rightarrow 3B = 9$$

$$\therefore B = 3$$

পুনরায় (ii) নং-এ, $x=2$ বসিয়ে পাই—

$$6\times 2 - 3 = A(2+1)(2-2) + B(2-2) + C(2+1)^2$$

$$\Rightarrow 12-3 = C\times 3^2$$

$$\Rightarrow 9C = 9$$

$$\therefore C = 1$$

(ii) নং-এ x^2 এর সহগ সমীকৃত করে পাই,

$$0 = A + C$$

$$\Rightarrow A = -C$$

$$\therefore A = -1$$

(i) নং-এ A, B, C এর মান বসিয়ে পাই,

$$\frac{6x-3}{(x+1)^2(x-2)} = \frac{-1}{x+1} + \frac{3}{(x+1)^2} + \frac{1}{x-2} \quad \text{(Ans)}$$

2. প্রমাণ কর যে, $1+\frac{2}{\underline{|1}}+\frac{3}{\underline{|2}}+\frac{4}{\underline{|3}}+\cdots\cdots\infty=2e$

সমাধানঃ এখানে, ধারাটি,

$$1+\frac{2}{\underline{|1}}+\frac{3}{\underline{|2}}+\frac{4}{\underline{|3}}+\cdots\cdots\infty$$

$$=\frac{1}{\underline{|0}}+\frac{2}{\underline{|1}}+\frac{3}{\underline{|2}}+\frac{4}{\underline{|3}}+\cdots\cdots\infty$$

সুতরাং n তম পদ,

$$T_n=\frac{n}{\underline{|n-1}}=\frac{(n-1)+1}{\underline{|n-1}}=\frac{n-1}{\underline{|n-1}}+\frac{1}{\underline{|n-1}}=\frac{n-1}{(n-1)\underline{|n-2}}+\frac{1}{\underline{|n-1}}$$

$$=\frac{1}{\underline{|n-2}}+\frac{1}{\underline{|n-1}}$$

এখন, $n=1,2,3\cdots$ ইত্যাদি বসিয়ে পাই,

$$T_1=\frac{1}{\underline{|1-2}}+\frac{1}{\underline{|1-1}}=\frac{1}{\underline{|-1}}+\frac{1}{\underline{|0}}$$

$$T_2=\frac{1}{\underline{|2-2}}+\frac{1}{\underline{|2-1}}=\frac{1}{\underline{|0}}+\frac{1}{\underline{|1}}$$

$$T_3=\frac{1}{\underline{|3-2}}+\frac{1}{\underline{|3-1}}=\frac{1}{\underline{|1}}+\frac{1}{\underline{|2}}$$

$$T_4=\frac{1}{\underline{|4-2}}+\frac{1}{\underline{|4-1}}=\frac{1}{\underline{|2}}+\frac{1}{\underline{|3}}$$

$$\cdots\cdots\cdots\cdots$$

প্রদত্ত ধারার যোগফল,

$$S=\left(\frac{1}{\underline{|-1}}+\frac{1}{\underline{|0}}+\frac{1}{\underline{|1}}+\frac{1}{\underline{|2}}\cdots\infty\right)+\left(\frac{1}{\underline{|0}}+\frac{1}{\underline{|1}}+\frac{1}{\underline{|2}}+\frac{1}{\underline{|3}}+\cdots\infty\right)$$

$$=\left(0+1+\frac{1}{\underline{|1}}+\frac{1}{\underline{|2}}+\cdots\infty\right)+\left(1+\frac{1}{\underline{|1}}+\frac{1}{\underline{|2}}+\frac{1}{\underline{|3}}+\cdots\infty\right)$$

$$=e+e \qquad \left[\because \underline{|-1}=\infty,\ \underline{|0}=1\right.$$

$$=2e$$

$\therefore$ L.H.S = R.H.S (Proved)

3. দেখাও যে, $\frac{1}{1!} + \frac{2}{3!} + \frac{3}{5!} + \cdots \infty = \frac{1}{2}e$

সমাধান: ধরি, $S = \frac{1}{1!} + \frac{2}{3!} + \frac{3}{5!} + \cdots \infty$

ধারাটির nth পদ $T_n = \frac{n}{(2n-1)!} = \frac{2n}{2(2n-1)!} = \frac{(2n-1)+1}{2(2n-1)!}$

$$= \frac{1}{2}\left[\frac{2n-1}{(2n-1)!} + \frac{1}{(2n-1)!}\right]$$

$$= \frac{1}{2}\left[\frac{2n-1}{(2n-1)(2n-2)!} + \frac{1}{(2n-1)!}\right]$$

$$= \frac{1}{2}\left[\frac{1}{(2n-2)!} + \frac{1}{(2n-1)!}\right]$$

$n = 1, 2, 3, 4, \ldots$ ইত্যাদি বসিয়ে পাই,

$T_1 = \frac{1}{2}\left[\frac{1}{(2\times1-2)!} + \frac{1}{(2\times1-1)!}\right] = \frac{1}{2}\left[\frac{1}{0!} + \frac{1}{1!}\right]$

$T_2 = \frac{1}{2}\left[\frac{1}{(2\times2-2)!} + \frac{1}{(2\times2-1)!}\right] = \frac{1}{2}\left[\frac{1}{2!} + \frac{1}{3!}\right]$

$T_3 = \frac{1}{2}\left[\frac{1}{(2\times3-2)!} + \frac{1}{(2\times3-1)!}\right] = \frac{1}{2}\left[\frac{1}{4!} + \frac{1}{5!}\right]$

যোগফল $S = T_1 + T_2 + T_3 + \cdots$

$$= \frac{1}{2}\left(\frac{1}{0!} + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \frac{1}{5!} + \cdots \infty\right)$$

$$= \frac{1}{2}e$$

$\therefore$ L.H.S = R.H.S (showed)

4. $1+\dfrac{1+2}{\underline{|2}}+\dfrac{1+2+3}{\underline{|3}}+\dfrac{1+2+3+4}{\underline{|4}}+\cdots\infty=\dfrac{3}{2}e$

<u>সমাধানঃ</u> $1+\dfrac{1+2}{\underline{|2}}+\dfrac{1+2+3}{\underline{|3}}+\dfrac{1+2+3+4}{\underline{|4}}+\cdots\infty$

ধরিঃ n^{th} তম পদ $T_n=\dfrac{1+2+3+\cdots n}{\underline{|n}}$

$$=\frac{n(n+1)}{2\underline{|n}}\qquad\left[\because 1+2+3+\cdots n=\frac{n(n+1)}{2}\right]$$

$$=\frac{n(n+1)}{2n\underline{|n-1}}$$

$$=\frac{n+1}{2\underline{|n-1}}$$

$$=\frac{(n-1)+2}{2\underline{|n-1}}$$

$$=\frac{n-1}{2\underline{|n-1}}+\frac{2}{2\underline{|n-1}}$$

$$=\frac{n-1}{2(n-1)\underline{|n-2}}+\frac{1}{\underline{|n-1}}$$

$$=\frac{1}{2\underline{|n-2}}+\frac{1}{\underline{|n-1}}$$

$n=1,2,3,4\cdots$ ইত্যাদি বসিয়ে পাই,

$$T_1=\frac{1}{2\underline{|1-2}}+\frac{1}{\underline{|1-1}}=\frac{1}{2}\cdot 0+1$$

$$T_2=\frac{1}{2\underline{|2-2}}+\frac{1}{\underline{|2-1}}=\frac{1}{2}\cdot 1+\frac{1}{\underline{|1}}$$

$$T_3=\frac{1}{2\underline{|3-2}}+\frac{1}{\underline{|3-1}}=\frac{1}{2}\cdot\frac{1}{\underline{|1}}+\frac{1}{\underline{|2}}$$

$$T_4=\frac{1}{2\underline{|4-2}}+\frac{1}{\underline{|4-1}}=\frac{1}{2}\cdot\frac{1}{\underline{|2}}+\frac{1}{\underline{|3}}$$

$\cdots\cdots\cdots$

$\cdots\cdots\cdots$

$\cdots\cdots\cdots$

প্রদত্ত ধারার যোগফল, $S = \frac{1}{2}\left(0+1+\frac{1}{\underline{|1}}+\frac{1}{\underline{|2}}+\cdots\infty\right)+\left(1+\frac{1}{\underline{|1}}+\frac{1}{\underline{|2}}+\frac{1}{\underline{|3}}+\cdots\infty\right)$

$$= \frac{1}{2}e + e$$

$$= \frac{e+2e}{2}$$

$$= \frac{3e}{2}$$

$$= \frac{3}{2}e$$

$\therefore L.H.S = R.H.S$ (Showed)

5. প্রমাণ কর যে, $\frac{2}{\underline{|1}} + \frac{2+4}{\underline{|2}} + \frac{2+4+6}{\underline{|3}} + \cdots\cdots = 3e$

<u>সমাধানঃ</u> ধারাটির n তম পদ $t_n = \frac{2+4+6+\cdots\cdots 2n}{\underline{|n}}$

$$= \frac{2(1+2+3+\cdots\cdots n)}{\underline{|n}}$$

$$= \frac{2\,n(n+1)}{2\underline{|n}}$$

$$= \frac{n(n+1)}{\underline{|n}}$$

$$= \frac{n(n+1)}{n\underline{|n-1}}$$

$$= \frac{(n-1)+2}{\underline{|n-1}}$$

$$= \frac{n-1}{\underline{|n-1}} + \frac{2}{\underline{|n-1}}$$

$$= \frac{n-1}{(n-1)\underline{|n-2}} + \frac{2}{\underline{|n-1}}$$

$$= \frac{1}{\underline{|n-2}} + \frac{2}{\underline{|n-1}}$$

এখন, $n = 1, 2, 3, 4$ ------ ইত্যাদি বসিয়ে পাই

$$t_1 = \frac{1}{\underline{|1-2}} + \frac{2}{\underline{|1-1}} = \frac{1}{\underline{|-1}} + \frac{2}{\underline{|0}}$$

$$t_2 = \frac{1}{\underline{|2-2}} + \frac{2}{\underline{|2-1}} = \frac{1}{\underline{|0}} + \frac{1}{\underline{|1}}$$

$$t_3 = \frac{1}{\underline{|3-2}} + \frac{2}{\underline{|3-1}} = \frac{1}{\underline{|1}} + \frac{2}{\underline{|2}}$$

$$t_4 = \frac{1}{\underline{|4-2}} + \frac{2}{\underline{|4-1}} = \frac{1}{\underline{|2}} + \frac{2}{\underline{|3}}$$

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প্রদত্ত ধারার যোগফল, $s = t_1 + t_2 + t_3 + t_4 + \cdots \infty$

$$\therefore s = \left(1 + \frac{1}{\underline{|1}} + \frac{1}{\underline{|2}} + \cdots \infty\right) + 2\left(1 + \frac{1}{\underline{|1}} + \frac{1}{\underline{|2}} + \frac{1}{\underline{|3}} + \cdots \infty\right)$$

$$= e + 2e$$
$$= 3e$$

$\therefore$ L.H.S = R.H.S (proved)

6. প্রমাণ করতে হবে যে, e একটি অসীম সংখ্যা এবং এর মান 2 অপেক্ষা বৃহত্তর এবং 3 অপেক্ষা ক্ষুদ্রতর।

প্রমাণঃ আমরা জানি,

$$e = 1 + \frac{1}{\underline{|1}} + \frac{1}{\underline{|2}} + \frac{1}{\underline{|3}} + \frac{1}{\underline{|4}} + \cdots \infty \quad \ldots (1)$$

$$= 2 + \frac{1}{\underline{|2}} + \frac{1}{\underline{|3}} + \frac{1}{\underline{|4}} + \cdots \infty \quad [\because \underline{|1} = 1]$$

সুতরাং $e > 2$

আবার, $\underline{|2} = 2 \cdot 1 = 2 \quad \therefore \frac{1}{\underline{|2}} = \frac{1}{2}$

$\underline{|3} = 3 \cdot 2 \cdot 1 > 2^2 \quad \therefore \frac{1}{\underline{|3}} < \frac{1}{2^2}$

$\underline{|4} = 4 \cdot 3 \cdot 2 \cdot 1 > 2^3 \quad \therefore \frac{1}{\underline{|4}} < \frac{1}{2^3}$

অনুরূপভাবে, $\frac{1}{\lfloor 5} < \frac{1}{2^4}$, $\frac{1}{\lfloor 6} < \frac{1}{2^5}$ ইত্যাদি

$$\therefore e = 1 + \frac{1}{\lfloor 1} + \frac{1}{\lfloor 2} + \frac{1}{\lfloor 3} + \frac{1}{\lfloor 4} + \cdots \infty < 1 + \left(1 + \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \cdots \infty\right)$$

$$e < 1 + \frac{1}{1 - \frac{1}{2}} = 1 + \frac{1}{\frac{2-1}{2}} = 1 + \frac{1}{1/2} = 1 + 2 = 3$$

অর্থাৎ $e < 3$

সুতরাং e এর মান 2 অপেক্ষা বৃহত্তর এবং 3 অপেক্ষা ক্ষুদ্রতর এবং

এটি একটি অসীম সংখ্যা,

$\therefore 2 < e < 3$ (প্রমাণিত)

7. $\left(2x^2 - \frac{1}{2x^3}\right)^{10}$ এর বিস্তৃতিতে x বর্জিত পদ ও এর মান নির্ণয় কর।

সমাধান: $\left\{2x^2 + \left(\frac{1}{-2x^3}\right)\right\}^{10}$

মনে করি, $(r+1)$ তম পদটি x বর্জিত।

$$(r+1) \text{ তম পদ} = {}^{10}c_r (2x^2)^{10-r} \cdot \left(-\frac{1}{2x^3}\right)^r$$

$$= {}^{10}c_r \cdot 2^{10-r} \cdot x^{20-2r} \cdot (-1)^r \cdot \frac{1}{2^r} \cdot x^{-3r}$$ 2^r

$$= (-1)^r \cdot {}^{10}c_r \cdot 2^{10-2r} \cdot x^{20-5r}$$

যেহেতু $(r+1)$ তম পদ x বর্জিত.

সুতরাং $x^{20-5r} = x^0$

$\Rightarrow 20 - 5r = 0$

$\Rightarrow 5r = 20$

$\therefore r = 4$

সুতরাং $r + 1 = 4 + 1 = 5$ তম পদ x বর্জিত, (Ans)

$\therefore$ নির্ণেয় মান $= (-1)^4 \cdot {}^{10}c_4 \cdot 2^{10-8}$

$= {}^{10}c_4 \cdot 2^2 = 840$ (Ans)

৪। $\left(2x - \frac{1}{4x^2}\right)^{12}$ এর বিস্তৃতিতে x বর্জিত পদ ও এর মান নির্ণয় কর।

সমাধান: $\left(2x - \frac{1}{4x^2}\right)^{12}$ $= \left\{2x + \left(\frac{1}{-4x^2}\right)\right\}^{12}$

মনে করি, $r+1$ তম পদ x বর্জিত,

$(r+1)$ তম পদ $= {}^{12}C_r \cdot (2x)^{12-r} \cdot \left(-\frac{1}{4x^2}\right)^r$

$= {}^{12}C_r \; 2^{12-r} \cdot x^{12-r} \cdot (-1)^r \cdot \frac{1}{4^r} \cdot \frac{1}{x^{2r}}$

$= {}^{12}C_r \cdot 2^{12-3r} \; (-1)^r \cdot x^{12-3r}$

যেহেতু $(r+1)$ তম x বর্জিত,

$12 - 3r = 0$

$\Rightarrow 3r = 12$

$\therefore r = 4$

সুতরাং $(r+1) = (4+1) = 5$ তম পদ x বর্জিত Ans

$\therefore$ নির্ণেয় মান: ${}^{12}C_4 \; 2^{12-12} \; (-1)^4$

$= {}^{12}C_4 \cdot 2^0 \cdot 1$

$= 495$ Ans

9. $(1+x)^{44}$ এর বিস্তৃতিতে 21 তম ও 22 তম পদ দুটি পরস্পর সমান হলে, x এর মান নির্ণয় কর।

সমাধান: এখানে, 21 তম পদ $= (20+1)$ তম পদ $= {}^{44}C_{20} \cdot x^{20}$

এবং 22 তম পদ $= (21+1)$ তম পদ $= {}^{44}C_{21} \cdot x^{21}$

প্রশ্নমতে,

$${}^{44}C_{20} \cdot x^{20} = {}^{44}C_{21} \cdot x^{21}$$

$$\Rightarrow \frac{|\underline{44}}{|\underline{20}\,|\underline{44-20}} = \frac{|\underline{44}}{|\underline{21}\,|\underline{44-21}} \cdot x$$

$$\Rightarrow \frac{1}{|\underline{20}\,|\underline{24}} = \frac{x}{21 \cdot |\underline{21-1}\,|\underline{23}}$$

$$\Rightarrow \frac{1}{|\underline{20}\; 24 \cdot |\underline{23}} = \frac{x}{21 \cdot |\underline{20} \cdot |\underline{23}}$$

$$\Rightarrow \frac{1}{24} = \frac{x}{21}$$

$$\Rightarrow x = \frac{21}{24} = \frac{7}{8} \quad \text{Ans}$$

10. দেখাও যে, $(1-2x)^{-\frac{1}{2}}$ এর বিস্তৃতিতে $(r+1)$ তম পদের সহগ $\dfrac{|\underline{2r}}{(|\underline{r})^2 \cdot 2^r}$

সমাধান: আমরা জানি,

$(1+x)^n$ এর বিস্তৃতিতে $(r+1)$ তম পদ $=$

$$\frac{n(n-1)(n-2)\,\cdots\cdots\,(n-r+1)}{|\underline{r}} \cdot x^r$$

$\therefore (1-2x)^{-1/2}$ এর বিস্তৃতিতে $(r+1)$ তম পদ,

$$\frac{\left(-\frac{1}{2}\right)\left(-\frac{1}{2}-1\right)\left(-\frac{1}{2}-2\right)\,\cdots\cdots\,\left(-\frac{1}{2}-r+1\right)}{|\underline{r}} \cdot (-2x)^r$$

$$= \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{5}{2}\right)\,\cdots\cdots\,\left(\frac{1-2r}{2}\right)}{|\underline{r}} (-1)^r \cdot 2^r \cdot x^r$$

$$= \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{5}{2}\right)\cdots\cdots\left(-\frac{2r-1}{2}\right)}{\lfloor r} \cdot (-1)^r \cdot 2^r \cdot x^r$$

$$= (-1)^r \cdot \frac{1}{2^r} \quad \frac{1 \cdot 3 \cdot 5 \cdots\cdots (2r-1)}{\lfloor r} \cdot (-1)^r \cdot 2^r \cdot x^r$$

$$= (-1)^{2r} \cdot \quad \frac{1 \cdot 3 \cdot 5 \cdots (2r-1)}{\lfloor r} \, x^r$$

$\therefore x^r$ এর সহগ $= \dfrac{1 \cdot 3 \cdot 5 \cdots (2r-1)}{\lfloor r}$

$$= \frac{1 \cdot 3 \cdot 5 \cdots (2r-1) \{2 \cdot 4 \cdot 6 \cdots (2r)\}}{\lfloor r \{2 \cdot 4 \cdot 6 \cdots (2r)\}}$$

$$= \frac{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdots (2r-1) \cdot 2r}{\lfloor r \{(2 \cdot 1) \cdot (2 \cdot 2) \cdot (2 \cdot 3) \cdots (2r)\}}$$

$$= \frac{\lfloor 2r}{\lfloor r \, (2 \cdot 2 \cdot 2 \cdots 2)\,(1 \cdot 2 \cdot 3 \cdots r)}$$

$$= \frac{\lfloor 2r}{\lfloor r \; 2^r \cdot \lfloor r}$$

$$= \frac{\lfloor 2r}{(\lfloor r)^2 \cdot 2^r}$$

(দেখানো হলো)

দেখাও যে, $(1-4x)^{-\frac{1}{2}}$ এর বিস্তৃতিতে $(r+1)$ তম পদ পদটির মান $\frac{\lfloor 2r}{(\lfloor r)^2} x^r$

<u>সমাধান</u>: আমরা জানি, $(r+1)$ তমপদ $= \frac{n(n-1)(n-2)\cdots(n-r+1)}{\lfloor r} \cdot x^r$

$\therefore (1-4x)^{-1/2}$ এর বিস্তৃতিতে $(r+1)$ তম পদ পদ,

$$= \frac{\left(-\frac{1}{2}\right)\left(-\frac{1}{2}-1\right)\left(-\frac{1}{2}-2\right)\cdots\left(-\frac{1}{2}-r+1\right)}{\lfloor r} (-4x)^r$$

$$= \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{5}{2}\right)\cdots \frac{2r-1}{2}}{\lfloor r} (-1)^r \cdot 4^r \cdot x^r$$

$$= (-1)^r \cdot \frac{1}{2^r} \cdot \frac{1\cdot 3\cdot 5 \cdots (2r-1)}{\lfloor r} (-1)^r \cdot 2^{2r} \cdot x^r$$

$$= (-1)^{2r} \cdot \frac{1\cdot 3\cdot 5 \cdots (2r-1)}{\lfloor r} \cdot 2^r \cdot x^r$$

$$= \frac{1\cdot 3\cdot 5 \cdots (2r-1)\{2\cdot 4\cdot 6 \cdots (2r)\}}{\lfloor r \{2\cdot 4\cdot 6 \cdots (2r)\}} \cdot 2^r \cdot x^r$$

$$= \frac{1\cdot 2\cdot 3\cdot 4\cdot 5\cdot 6 \cdots 2r}{\lfloor r \{(2\cdot 1)(2\cdot 2)(2\cdot 3)\cdots 2\cdot r\}} \cdot 2^r \cdot x^r$$

$$= \frac{\lfloor 2r}{\lfloor r \cdot 2^r \cdot (1\cdot 2\cdot 3 \cdots r)} \cdot 2^r \cdot x^r$$

$$= \frac{\lfloor 2r}{\lfloor r \cdot 2^r \cdot \lfloor r} \cdot 2^r \cdot x^r$$

$$= \frac{\lfloor 2r}{(\lfloor r)^2} \cdot x^r$$

(দেখানো হলো)

12. $\sin^2(\log \sec x)$

সমাধানঃ $\frac{d}{dx} \sin^2(\log \sec x)$

$= 2[\sin(\log \sec x)]^{2-1}$

$= 2\sin(\log \sec x) \cdot \cos(\log \sec x) \cdot \frac{d}{dx}(\log \sec x)$

$= \sin\{2(\log \sec x)\} \cdot \frac{1}{\sec x} \cdot \frac{d}{dx}(\sec x)$ $[2\sin A\cos A = \sin 2A]$

$= \sin\{2(\log \sec x)\} \cdot \frac{1}{\sec x} \cdot \sec x \cdot \tan x$

$= \sin(2\log \sec x) \cdot \tan x$

$= \sin(\log \sec^2 x) \cdot \tan x$ (Ans)

13. $\frac{1+\sin x}{1-\sin x}$

সমাধানঃ $\frac{d}{dx}\left(\frac{1+\sin x}{1-\sin x}\right)$

$= \frac{(1-\sin x)\frac{d}{dx}(1+\sin x) - (1+\sin x)\frac{d}{dx}(1-\sin x)}{(1-\sin x)^2}$

$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{d}{dx}u - u\frac{d}{dx}v}{v^2}$

$= \frac{(1-\sin x)\cos x - (1+\sin x)(-\cos x)}{(1-\sin x)^2}$

$= \frac{\cos x(1-\sin x + 1+\sin x)}{(1-\sin x)^2}$

$= \frac{2\cos x}{(1-\sin x)^2}$ (Ans)

14. $\dfrac{\sin x + \cos x}{\sqrt{1+\sin 2x}}$

সমাধানঃ ধরি, $y = \dfrac{\sin x + \cos x}{\sqrt{1+\sin 2x}}$

$$= \frac{\sin x + \cos x}{\sqrt{1 + 2\sin x \cos x}} = \frac{\sin x + \cos x}{\sqrt{(\sin x + \cos x)^2}} = 1$$

$$\Rightarrow \frac{dy}{dx} = \frac{d}{dx}(1)$$

$$= 0 \text{ (Ans)}$$

15. $\tan^{-1}\dfrac{\sqrt{1+x^2}-1}{x}$

সমাধানঃ ধরি, $y = \tan^{-1}\dfrac{\sqrt{1+x^2}-1}{x}$ এবং $x = \tan\theta$

$\therefore \theta = \tan^{-1}x$

$$\Rightarrow y = \tan^{-1}\frac{\sqrt{1+\tan^2\theta}-1}{\tan\theta}$$

$$= \tan^{-1}\frac{\sec\theta - 1}{\tan\theta}$$

$$= \tan^{-1}\frac{\frac{1}{\cos\theta}-1}{\frac{\sin\theta}{\cos\theta}} = \tan^{-1}\frac{1-\cos\theta}{\cos\theta} \times \frac{\cos\theta}{\sin\theta}$$

$$= \tan^{-1}\frac{1-\cos\theta}{\sin\theta} = \frac{2\sin^2\theta/2}{2\sin\theta/2 \cos\theta/2}$$

$$= \tan^{-1}\tan\theta/2$$

$$= \frac{1}{2}\theta$$

$$= \frac{1}{2}\tan^{-1}x$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2}\frac{d}{dx}\tan^{-1}x$$

$$= \frac{1}{2}\cdot\frac{1}{1+x^2}$$

$$= \frac{1}{2(1+x^2)} \text{ (Ans)}$$

16. $y = x^2 + \sqrt{1-x^2}$ বক্ররেখার উপর যে-সব বিন্দুতে স্পর্শক x অক্ষের উপর লম্ব সেসব বিন্দুসমূহের স্থানাঙ্ক বের কর।

সমাধানঃ দেওয়া আছে,

$$y = x^2 + \sqrt{1-x^2} \quad \text{--------} \quad (i)$$

$$\Rightarrow \frac{dy}{dx} = 2x + \frac{1}{2\sqrt{1-x^2}} \times \frac{d}{dx}(1-x^2)$$

$$\Rightarrow \frac{dy}{dx} = 2x + \frac{1}{2\sqrt{1-x^2}} \times (-2x) = 2x - \frac{x}{\sqrt{1-x^2}}$$

$$= \frac{2x\sqrt{1-x^2} - x}{\sqrt{1-x^2}}$$

যেহেতু স্পর্শক x অক্ষের উপর লম্ব বা y অক্ষের সমান্তরাল -

সুতরাং $\frac{dy}{dx} = \infty$

$$\Rightarrow \frac{2x\sqrt{1-x^2} - x}{\sqrt{1-x^2}} = \infty$$

$$\Rightarrow \sqrt{1-x^2} = 0 \qquad \left[\because \infty = \frac{1}{0}\right]$$

$$\Rightarrow 1 - x^2 = 0$$

$$\Rightarrow x^2 = 1$$

$$\therefore x = \pm 1$$

(1) নং-এ x এর মান বসিয়ে পাই -

যখন $x = 1$ তখন $y = 1^2 + \sqrt{1-1^2} = 1 + 0 = 1$

" $x = -1$ " $y = (-1)^2 + \sqrt{1-(-1)^2} = 1 + 0 = 1$

$\therefore$ নির্ণেয় বিন্দুসমূহের স্থানাঙ্ক $(1, 1)$ এবং $(-1, 1)$ A

17. যদি $u = \cos^{-1}\frac{x}{y} + \tan^{-1}\frac{y}{x}$ হয়, তবে প্রমাণ কর যে, $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 0$

<u>সমাধান:</u> দেওয়া আছে,

$$u = \cos^{-1}\frac{x}{y} + \tan^{-1}\frac{y}{x}$$

$$\Rightarrow \frac{\partial u}{\partial x} = -\frac{1}{\sqrt{1-\frac{x^2}{y^2}}} \times \frac{1}{y} + \frac{1}{1+\frac{y^2}{x^2}} \times \left(-\frac{y}{x^2}\right)$$

$$\boxed{\begin{array}{l} \frac{d}{dx}\left(\frac{x}{y}\right) = \frac{1}{y} \\ \frac{d}{dx}\left(\frac{y}{x}\right) = \frac{-y}{x^2} \end{array}}$$

$$= -\frac{1}{\sqrt{\frac{y^2-x^2}{y^2}}} \times \frac{1}{y} - \frac{y}{\frac{x^2+y^2}{x^2}} \times \frac{1}{x^2}$$

$$= -\frac{y}{\sqrt{y^2-x^2}} \times \frac{1}{y} - \frac{x^2 y}{x^2+y^2} \times \frac{1}{x^2}$$

$$= -\frac{1}{\sqrt{y^2-x^2}} - \frac{y}{x^2+y^2}$$

$$\therefore x\frac{\partial u}{\partial x} = \frac{-x}{\sqrt{y^2-x^2}} - \frac{xy}{x^2+y^2} \text{ ------ (i)}$$

আবার, $$\frac{\partial u}{\partial y} = \frac{-1}{\sqrt{1-\frac{x^2}{y^2}}} \times -\frac{x}{y^2} + \frac{1}{1+\frac{y^2}{x^2}} \times \frac{1}{x}$$

$$\boxed{\begin{array}{l} \frac{d}{dy}\left(\frac{x}{y}\right) = -\frac{x}{y^2} \\ \frac{d}{dy}\left(\frac{y}{x}\right) = \frac{1}{x} \end{array}}$$

$$= \frac{-1}{\sqrt{\frac{y^2-x^2}{y^2}}} \times -\frac{x}{y^2} + \frac{1}{\frac{x^2+y^2}{x^2}} \times \frac{1}{x}$$

$$= \frac{-y}{\sqrt{y^2-x^2}} \times -\frac{x}{y^2} + \frac{x^2}{x^2+y^2} \times \frac{1}{x}$$

$$= \frac{x}{y\sqrt{y^2-x^2}} + \frac{x}{x^2+y^2}$$

$$\therefore y\frac{\partial u}{\partial y} = \frac{x}{\sqrt{y^2-x^2}} + \frac{xy}{x^2+y^2} \text{ ------ (ii)}$$

(i) + (ii) করে পাই —

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{-x}{\sqrt{y^2-x^2}} - \frac{xy}{x^2+y^2} + \frac{x}{\sqrt{y^2-x^2}} + \frac{xy}{x^2+y^2}$$

$$\therefore x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 0$$

(প্রমাণিত)

18. $\int \sin 5x \sin 3x \, dx$

সমাধানঃ $\int \sin 5x \sin 3x \, dx$

$= \frac{1}{2}\int 2 \sin 5x \sin 3x \, dx$

$= \frac{1}{2}\int \{\cos(5x-3x) - \cos(5x+3x)\} dx$

$[\because 2 \sin A \sin B = \cos(A-B) - \cos(A+B)]$

$= \frac{1}{2}\int (\cos 2x - \cos 8x) dx$

$= \frac{1}{2}\int \cos 2x \, dx - \frac{1}{2}\int \cos 8x \, dx$

$= \frac{1}{2} \frac{\sin 2x}{2} - \frac{1}{2} \frac{\sin 8x}{8} + c$

$[\because \int \cos ax = \frac{1}{a} \sin ax]$

$= \frac{1}{4} \sin 2x - \frac{1}{16} \sin 8x + c$ (Ans)

19. $\int e^{\tan^{-1}x} \cdot \frac{1}{1+x^2} dx$

সমাধানঃ $\int e^{\tan^{-1}x} \cdot \frac{1}{1+x^2} dx$

$= \int e^z dz$

$= e^z + c$

$= e^{\tan^{-1}x} + c$ (Ans)

ধরি,

$z = \tan^{-1}x$

$\Rightarrow \frac{dz}{dx} = \frac{1}{1+x^2}$

$\therefore dz = \frac{1}{1+x^2} dx$

$$\int_0^1 x^3\sqrt{1+3x^4}\,dx$$

সমাধান: $\int_0^1 x^3\sqrt{1+3x^4}\,dx$

ধরি,

$1+3x^4 = z$

$\Rightarrow 0+12x^3\,dx = dz$

$\Rightarrow x^3\,dx = \frac{1}{12}\,dz$

যখন $x=0$, তখন $z=1$

" $x=1$, তখন $z=4$

$$=\int_1^4 \frac{1}{12}\sqrt{z}\,dz$$

$$=\frac{1}{12}\int_0^4 z^{1/2}\,dz$$

$$=\frac{1}{12}\left[\frac{z^{3/2}}{3/2}\right]_1^4$$

$$=\frac{1}{18}\left[4^{3/2}-(1)^{3/2}\right]$$

$$=\frac{1}{18}(8-1)$$

$$=\frac{1}{18}\times 7$$

$$=\frac{7}{18} \text{ (Ans)}$$

21. $$\int_0^{\log 2}\frac{e^x}{1+e^x}\,dx$$

সমাধান: $\int_0^{\log 2}\frac{e^x}{1+e^x}\,dx$

ধরি,

$1+e^x = z$

$\Rightarrow 0+e^x\,dx = dz$

$\Rightarrow e^x\,dx = dz$

যখন, $x=\log 2$, তখন, $z = 1+e^{\log 2}$

$= 1+2e^{\log}$

$= 1+2 = 3$

যখন $x=0$, তখন, $z = 1+e^0$

$=1+1$

$=2$

$$=\int_2^3 \frac{1}{z}\,dz$$

$$=\left[\log z\right]_2^3$$

$$=\log 3-\log 2$$

$$=\log\frac{3}{2} \text{ (Ans)}$$

22. $\int_0^{\sqrt{3}} \frac{(\tan^{-1}x)^2}{1+x^2}dx$

সমাধান: $\int_0^{\sqrt{3}} \frac{(\tan^{-1}x)^2}{1+x^2}dx$

ধরি,

$z = \tan^{-1}x$

$\therefore dz = \frac{1}{1+x^2}dx$

যখন, $x = \sqrt{3}$, তখন $z = \tan^{-1}\sqrt{3}$
$= \pi/3$

" $x = 0$ তখন $z = \tan^{-1}0$
$= 0$

$= \int_0^{\pi/3} z^2 dz$

$= \left[\frac{z^3}{3}\right]_0^{\pi/3}$

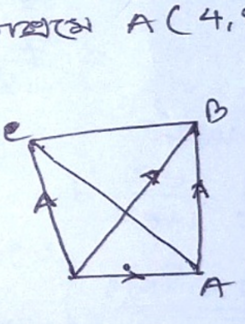
$= \frac{1}{3}\left[(\pi/3)^3 - 0\right]$

$= \frac{1}{3} \times \frac{\pi^3}{3^3}$

$= \frac{\pi^3}{81}$ Ans

23. একটি ত্রিভুজের শীর্ষবিন্দুর অবস্থান ভেক্টরগুলো যথাক্রমে (4, 5, 1), (2, 4, -1) ও (3, 6, -3) হলে দেখাও যে, এটি একটি সমকোণী সমদ্বিবাহু ত্রিভুজ।

সমাধান: ধরি ত্রিভুজটির প্রদত্ত শীর্ষবিন্দু তিনটি যথাক্রমে A(4, 5, 1), B(2, 4, -1) এবং C(3, 6, -3) এবং মূলবিন্দু O।



$\therefore \vec{OA} = 4\hat{i} + 5\hat{j} + \hat{k}$, $\vec{OB} = 2\hat{i} + 4\hat{j} - \hat{k}$

এবং $\vec{OC} = 3\hat{i} + 6\hat{j} - 3\hat{k}$

এখন, $\vec{AB} = \vec{OB} - \vec{OA} = 2\hat{i} + 4\hat{j} - \hat{k} - 4\hat{i} - 5\hat{j} - \hat{k} = -2\hat{i} - \hat{j} - 2\hat{k}$

$\therefore AB = |\vec{AB}| = \sqrt{(-2)^2 + (-1)^2 + (-2)^2} = \sqrt{4+1+4} = \sqrt{9} = 3$

আবার, $\vec{BC} = \vec{OC} - \vec{OB} = 3\hat{i} + 6\hat{j} - 3\hat{k} - 2\hat{i} - 4\hat{j} + \hat{k} = \hat{i} + 2\hat{j} - 2\hat{k}$

$\therefore BC = |\vec{BC}| = \sqrt{1^2 + 2^2 + (-2)^2} = \sqrt{1+4+4} = \sqrt{9} = 3$

এবং $\vec{AC} = \vec{OC} - \vec{OA} = 3\hat{i} + 6\hat{j} - 3\hat{k} - 4\hat{i} - 5\hat{j} - \hat{k} = -\hat{i} + \hat{j} - 4\hat{k}$

$\therefore AC = |\vec{AC}| = \sqrt{(-1)^2 + (1)^2 + (-4)^2} = \sqrt{1+1+16} = \sqrt{18} = 3\sqrt{2}$

ΔABC এ $AB = BC$ এবং $AB^2 + BC^2 = AC^2$

সুতরাং ত্রিভুজটি একটি সমকোণী সমদ্বিবাহু ত্রিভুজ।